Mathematics

Mathematical basics

We said that physics is the science of observing dependence of events in nature and translating them into clear, quantitative laws. Those laws are described using parameters. A parameter is a quantity of the system that quantitatively expresses the selected features of the system. Thus, physical laws express the dependence of parameters of one or more observed systems. For the description of laws, we use a mathematical vocabulary, and with mathematics, one can describe those dependences using functions. A function is a mathematical procedure that joins quantities from one set with quantities from a second set. For example, if the body is moving, the parameters are: path (distance), speed, and time. One can say that the path (distance) is a function of speed and time. The law of uniform motion along a straight line connects these parameters and shows dependence as a linear function (s = vt). So, if we want to apply some physical law to a biological system, we must know the parameters of the system and the meaning of mathematical functions that connect them. Therefore, in this mathematical introduction, we will consider mathematical models that can be applied in medicine.

The general form of writing functions is: $y = f(x, t)$. y is the dependent variable (or parameter) and x is independent variable. Process f describes how y is changing when x or t changes. Functions can be displayed graphically in the coordinate system, in a table where one set of numbers is associated with a second set of numbers, and analytically, using mathematical expression.

Linear function

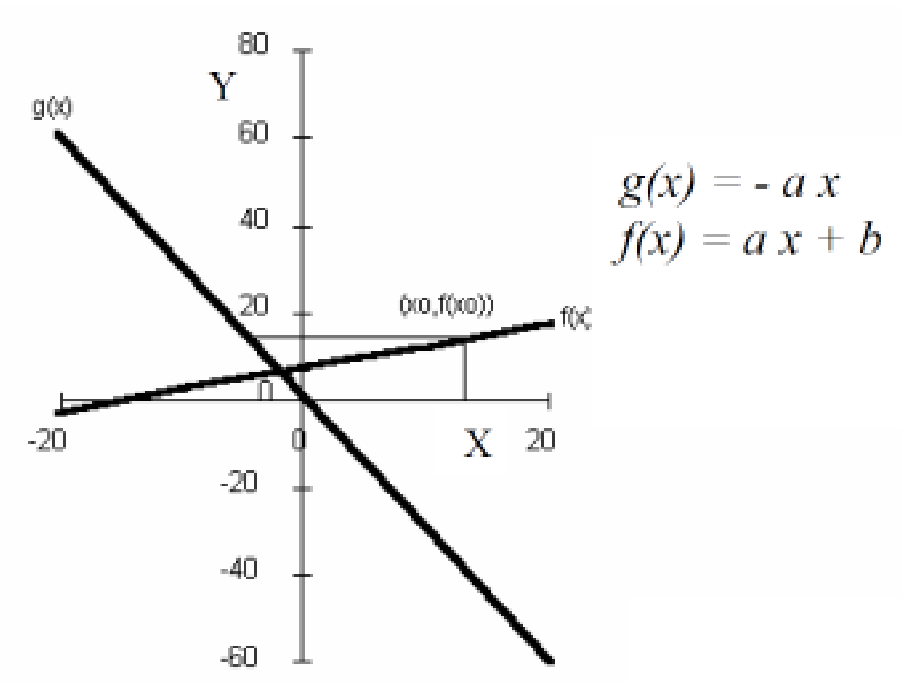

Figure 1. Linear function: g (x) is a decreasing; f (x) is an increasing

Mathematical expression $y = ax + b$ defines the linear dependence of parameter *y* on value *x*. The graphic description of the linear function in the X,Y coordinate system is a straight line through the origin or through value *b* on the *y* axis, with a direction coefficient of *a*. Values *x* and *y* are linearly dependent - when there is an increase or decrease of the parameter *x*, there is a proportional increase or decrease of the parameter *y*. We can also say that they are proportional parameters. If *a* <0, the function is decreasing (*g (x)* in figure 1). The function $y = ax + b$ is graphically displayed by a straight line, with the direction coefficient *a* and a shift on the y-axis, *b* (*f (x)* in figure 1). Dependence expressed this way means that the function has a value of b when the value of the independent parameter (*x*) is equal to 0. In physics, we say that *b* is the initial value of function.

The function of reciprocal dependence

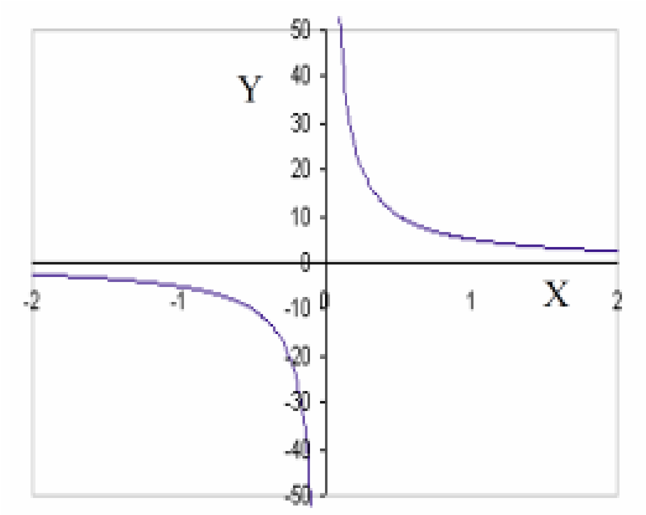

Figure 2 equilateral hyperbole

If the system parameters are related in a way that the increase of one parameter results in the decrease of another, we talk about reciprocal relationship. This dependence is shown using the function *y = a/x*. The graphic representation of the function is an equilateral hyperbole. Its asymptotes are coordinate axes X and Y.

Exponential function

The exponential dependence of the quantity *y* on quantity *x* is generally shown with expression $y = y_0 a^{bx}$. The parameters are: y_0 is the value of the function when *x* = 0, *a* is the base of the function, and *b* is a coefficient that determines the slope and flow of the curve. The curve intersects the Y axis at the point (0, y_0). In physics, this value is called the initial value of the function, i.e. the initial condition. The X-axis is the asymptote of the curve $x \Rightarrow -\infty$ if b> 0. That is, an increasing exponential function. A decreasing exponential function has b <0 and the curve is then asymptotically approaching the X axis with $x \Rightarrow +\infty$.

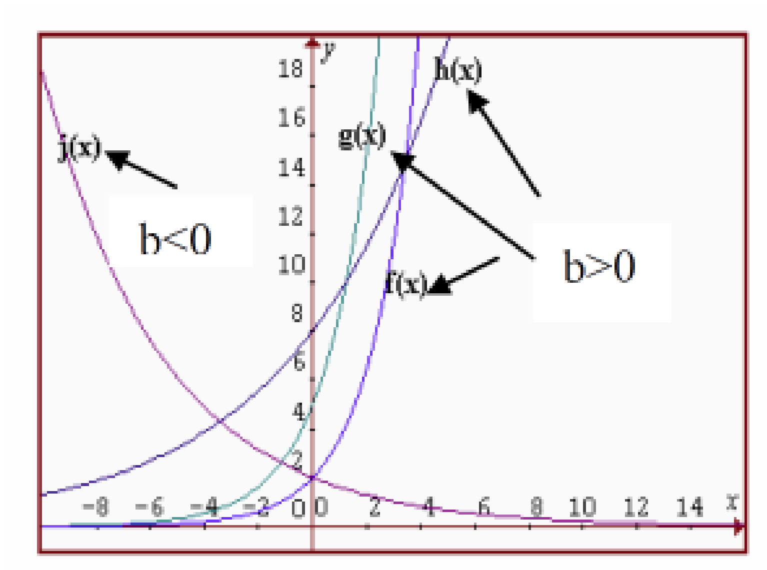

Figure 3 Exponential function f (x), g (x) and h (x) are growing exponential functions with different values of y_0 and a positive coefficient b; j (x) is a decreasing exponential function with a negative coefficient b.

Of particular importance in physics and biology is the exponential function with a base equal to the natural logarithm, $y = y_0 e^{bx}$. An important property of this function is that at every point increment, the function is proportional to its value, $dy/dx \sim y$, and therefore, it is a good description of the natural processes when individual events are random. We will mention two examples. Measurements have shown that increasing exponential functions, $N = N_0 2^{bt}$, describe bacterial growth well. Decreasing exponential functions can demonstrate the absorption of a drug in the body, $N = N_0 2^{bt}$. In both cases, N_0 is the value of the function at the beginning of the observation. The coefficients b and c depend on the rate of bacterial reproduction or the rate of drug absorption in the body.

Logarithmic function

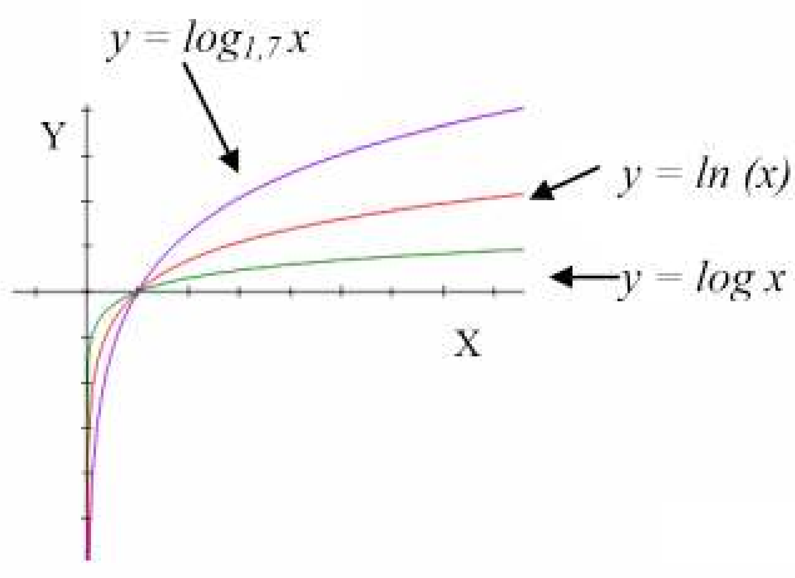

Figure 4 Logarithmic dependence

The logarithmic function is the inverse function of the exponential function. The function has the form $x = a^y$. If a = e then *y = ln x*, and if a = 10, the function becomes y = log x. These are the most common logarithmic functions, but the base of the function can have any value.

The logarithmic relationship of the parameters is not the usual relationship used in describing biological systems. The exception to the rule is the logarithmic relationship of the body's response to a stimulus. For example, the relationship between the sense of sound and the intensity of mechanical waves is shown using $G = l \log I$. In general, the logarithmic function is typically applied for a change of the parameters with a wide range of values.

Periodic function

Some events in nature are repeated at a specific time and/or space intervals. The state of a system is repeating itself in a correct and constant sequence like day giving rise to night, the cycle of seasons, the heart's heartbeat, or respiration. Examples of more regular periodic events include vibrations, circular motion, or waves. Functions describing such events must also satisfy the requirement of repetition. We're talking about periodic functions.

In general, we say that a function is periodic if it meets the basic requirement of f (x + X) = f (x). This means that after every change of the independent variable for X, the value of function repeats. Interval X is a period of periodic functions. If the independent parameter of periodic changes is time, as in case of vibration, then we call it time period, T, of periodic functions. It's the smallest time after which the function value will be repeated. For vibration that is the time of one oscillation. Reciprocal value of the time period is the frequency $\nu = 1 / T$, or the number of oscillations per second. Current value of the function, A_t, is the elongation and its maximum value of the elongation, A_0, is the amplitude. Vibration amplitude is the greatest distance from the equilibrium position. If the change of elongation in an observed time interval can be described using the *sin* function, then such vibration can be called harmonic oscillation. The term that describes the variation of elongation of harmonic oscillation is:

$$A_t = A_0 \sin \frac{2\pi t}{T} \quad ili \quad A_t = A_0 \sin 2\pi \nu t$$

If we look at the movement around a circle, we note that during one period, the radius vector will sweep over the angle 2π. The quantity that we introduce as a parameter for this motion is the angular velocity or angular frequency $\omega = 2\pi/T = 2\pi\nu$. Thus, we can now describe the variation of vibrational elongation as:

$$A_t = A_0 sin\omega t$$

For those functions for which the periodicity is observed in the dimension of space, the spatial period is defined as wavelength λ. It is the shortest distance between two points in space where the value of the function is the same or which oscillates in phase. An example of such periodicity is the wave motion. Harmonic wave is a simple periodic phenomena in space and time and is shown sinusoidal function

$$A(x,t) = A_0 \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Periodic functions that describe natural events are usually much more complex than sinusoidal functions. We're talking about anharmonic functions. Anharmonic functions describe all physiological processes, electrical and magnetic impulses that are measured in the diagnostics, and voices. Anharmonic functions cannot be described by analytical expression, so they are mathematically described using the Fourier theorem. Anharmonic function is displayed as a series of finite number of harmonic functions:

$$y(t) = \sum_{i=1}^{\infty} y_i \sin \omega_i t$$

The first member of the series is called the first harmonic and its frequency ω_1 is equal to the frequency of the anharmonic function. Other members of the series are higher harmonics and their frequencies are multiples of the frequency of the first harmonic: $\omega_n = n\omega_1$ (n = 2,3 ..).

Physical quantities

Physical quantities are either scalars or vectors. Size, which is completely determined only by the amount, is scalar. For example, scalars are mass, energy density, concentration, etc. Calculations with scalars use ordinary mathematical operations, as used in calculations with real numbers.
The vector quantity is a parameter fully described by both a magnitude and a direction. Vectors are: time, speed, acceleration, force, etc. Vector operations are not as simple as operations with numbers. Vectors are usually shown with their projections on the coordinate axes. Arithmetic operation is then performed using the projections. The result of the addition and subtraction of the vectors is always a vector. Rules of graphical addition of vectors are known rules: the rule of triangles and the polygon rule. The result of the multiplication of vectors can be either scalar or vector quantity, so we describe the scalar and vector product.
Scalar (dot) product of two vectors is a scalar quantity that has an amount equal to the product of the amount of the multiplied vectors and the cosine of the angle between them.

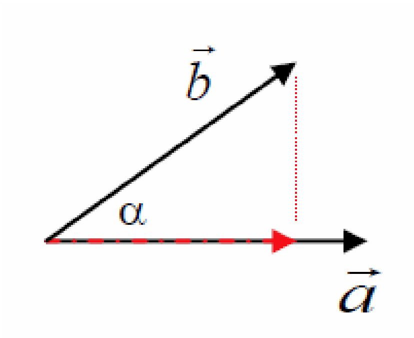

Figure 5 Scalar product $\vec{a}\ and\ \vec{b}\ is\ S = \vec{a} \cdot \vec{b} = ab\ sin\ \alpha$

Work is the scalar product of force and time: $W = \vec{F} \cdot \vec{s} = s\ Fcos\ \alpha$. Expression for the scalar product shows which part of the force is active in this work (the projection of force on the way).
The vector product of two vectors is the vector with a direction perpendicular to the plane, specified with multiplied vectors. The orientation is determined by the rule of the right hand, with the amount equal to the product of the amount of vectors and sine of the angle between them.

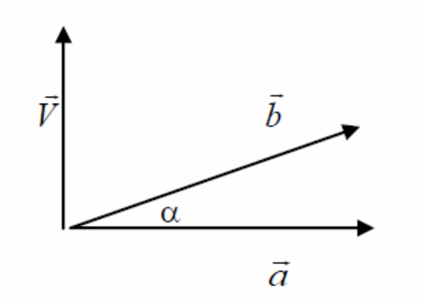

Figure 6 Vector product of vectors $\vec{a}\ and\ \vec{b}\ \vec{V} = \vec{a} \times \vec{b}$ Amount is $V = a\ b\ sin\ \alpha$

Force, with which magnetic field B affects the electrical current, I, in the conductor with a length, l, is the vector product of the magnetic induction and current $\vec{F} = I\vec{l} \times \vec{B}$. The direction of the carrier force is perpendicular to the plane defined by the vector of the magnetic induction and the direction of the current in the conductor. The amount of the force is equal to the surface parallelogram that is determined using these vectors. The orientation of force is designated by the rule of right hand.